Design of Optimum Structures to Impulse Type Loading

V. B. Venkayya* and N. S. Khot*
Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio

The purpose of this paper is to present the results of an analytical study on optimization problems involving large structures subjected to impulse type loading. The impact due to aircraft landing, aerodynamic gust, and the effect of blast waves are a few examples of impulse-type loadings on aerospace structures. The method of optimization presented in this paper is called designing in the dynamic mode. The dynamic mode may be a single natural mode of the structure or a combination of a set of natural modes depending on the spatial distribution and dynamic characteristics of the forcing function. The aperiodic forcing function is represented by Fourier integral in determining the dynamic response. A procedure for determining the participating modes in the dynamic response is discussed. The illustrative examples are a wing structure and a circular arch.

I. Introduction

OPTIMIZATION and automated design of structures has been the subject of intense study in recent years. Optimization has a dual function in design. It not only improves the merit function but often provides insight into behavior with respect to alternate (or possible) load paths. There are a number of structural optimization programs in operation with industry and Government. The notable advances to date are in the area of minimum weight design of structures subjected to static loads with stress, displacement, and size constraints.[1-5] Computer programs are available in this area for design of structures with several hundred or even several thousand degrees of freedom and comparable number of design variables. Efforts are underway at present to extend these capabilities to include aeroelastic requirements and designs for periodic and aperiodic forces.[6-8]

The purpose of this paper is to present an optimization method and its application to design problems involving structures subjected to impulse type loading. Impact due to aircraft landing, aerodynamic gusts, and the effect of blast waves are a few examples of impulse-type loading on aerospace structures. The optimization approach discussed in this paper is called "designing in the dynamic mode." The dynamic mode may consist of a single natural mode or a combination of a set of natural modes depending on the spatial distribution and dynamic characteristics of the forcing function. The theoretical development is addressed to general aperiodic forcing functions.

In the past, many optimization studies treated the design for dynamic loads as being essentially a frequency constrained problem. This hypothesis is acceptable when the forcing function is such that it activates only a single natural mode. In general, structural response involves several natural modes and as such dynamic optimization must be considered as a response constrained problem. The particular modes that participate in the dynamic response depend to a large extent on the spatial distribution of the forcing function. The dynamic stresses and the sizes of the elements are treated as the constraints for the examples presented in this paper.

II. Dynamic Analysis

The interest of the present paper is primarily in built-up structures, and the continuum is approximated by a discretized finite element model. Even though the procedure is developed in the context of the displacement method it can be used just as easily with the force method or the method of finite differences. The strain energy and kinetic energy of an element of a dynamic system are given by

$$u_i = 1/2 \int_{v_i} \sigma_i{}^t e_i \, dv \tag{1}$$

$$\tau_i = 1/2 \int_{v_i} \rho_i \dot{w}_i^t \dot{w}_i \, dv \tag{2}$$

where σ and e are the stress and strain vectors in the element; ρ and v are the mass density and the volume of the structural element. The vector w defines the displacement field in the element and is a continuous function of the spatial coordinates.

The displacement field may be represented by a set of discrete generalized coordinates of the i^{th} element in the form

$$w_i = \phi_i v_i \tag{3}$$

where v is the vector of discrete generalized coordinates, and ϕ is a rectangular matrix whose elements are functions of the spatial coordinates. The strain displacement relations can be expressed as

$$e_i = B\, w_i \tag{4}$$

where B is the differential operator.

In case of a linearly elastic material, the stresses and strains are related by

$$\sigma_i = G_i\, e_i \tag{5}$$

where G is the matrix of elastic constants. Substitution of Eqs. (3-5) into Eqs. (1) and (2) give the expressions for strain and kinetic energies of the element in the following form

$$u_i = 1/2\; v_i^t\, k_i\, v_i \tag{6}$$

$$\tau_i = 1/2\; \dot{v}_i^t\, m_i\, \dot{v}_i \tag{7}$$

where

$$k_i = \int_{v_i} \phi_i^t\, B_i^t\, G_i\, B_i\, \phi_i \, dv \tag{8}$$

$$m_i = \int_{v_i} \rho_i\, \phi_i^t\, \phi_i \, dv \tag{9}$$

are element stiffness and mass matrices, respectively.

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*Aerospace Engineer, Structures Division. Member AIAA.

The generalized coordinates of the elements and the structure are related by

$$v_i = a_i\, r \tag{10}$$

where r is the vector of system generalized coordinates and it represents the actual dynamic response of the structure, and a is the element to structure compatibility matrix. The expressions for the total energies of the structure are obtained by summing the component energies

$$U = 1/2\; r^t\, K\, r \tag{11}$$

$$T = 1/2\; \dot{r}^t\, M\, \dot{r} \tag{12}$$

where

$$K = \sum^{n} a_i^t\, k_i\, a_i \tag{13}$$

$$M = \sum^{n} a_i^t\, m_i\, a_i + M_c \tag{14}$$

are the generalized stiffness and mass matrices of the structure. n represents number of elements in the structure.

The second term in Eq. (14) represents concentrated or lumped masses due to nonstructural attachments such as pylons and fuel tanks in airplane wings. Substitution of Eqs. (11) and (12) into Lagrange's Equation gives the equations of motion of the system in the form

$$M\, \ddot{r} + K\, r = R(t) \tag{15}$$

The solution for the homogeneous part of Eq. (15) becomes the standard eigenvalue problem

$$\omega^2\, M\, \psi = K\, \psi \tag{16}$$

where ω and ψ represent the natural frequency and natural mode of the structure. The mass and stiffness matrices are both symmetric and are, in general, positive definite. In addition, these matrices are sparsely populated. To take advantage of these properties, the Sturm sequence property in conjunction with a bisection procedure is used to determine the eigenvalues. The eigenmodes are determined by inverse iteration.

The particular solution of Eq. (15) is obtained by a finite number of normal coordinates in the form

$$r = \psi\, q \tag{17}$$

where each column of matrix ψ is a normal mode and q represents the vector of normal coordinates. Substitution of Eq. (17) into (15) yields a set of uncoupled equations in the form

$$\ddot{q}_i + \omega^2\, q_i = (1/M_i)\, \psi_i^t\, R_o\, F(t) \tag{18}$$

where M_i is the generalized mass in the i^{th} mode. It is given by

$$M_i = \psi_i^t\, M\, \psi_i \tag{19}$$

where R_o is the generalized force vector, and its elements are functions of the spatial coordinates only. The time function $F(t)$ is assumed to be the same for all the generalized forces. If this is not true, the response due to each time function can be determined separately, and the combined response can be obtained by superposition.

The number of normal modes selected to approximate the dynamic response determines the number of uncoupled equations [Eq. (18)]. If all the normal modes are included in Eq. (17), the response would be exact to the degree of approximation expected of a discrete analysis. However, in practice, only a small number of modes would be necessary to obtain a reasonable estimate of the response. The particular modes and the number required depends primarily on the spatial distribution of the forcing function.

The solution of the uncoupled equation [Eq. (18)] can be obtained by the use of the convolution integral and it is written in the form[9]

$$q_i(t) = (1/\omega_i^2 M_i)\, \psi^t\, R_o\, \mathrm{D}_i\,(t) \tag{20}$$

where $D_i(t)$ is the dynamic load factor or magnification factor. This is given by

$$D_i(t) = \int_o^t \omega_i\, \mathrm{Sin}\; \omega_i(t-\tau)\; F(\tau)\; \mathrm{d}\tau \tag{21}$$

where t represents the time at which the response is desired and τ is the intermediate variable.

Substitution of Eq. (20) into (17) gives the actual response of the structure in the following form:

$$r = \psi\, D\, \psi^t\, R_o \tag{22}$$

where the diagonal matrix D is called the dynamic load factor matrix and its elements are given by

$$D_{ii} = D_i(t)/\omega_i^2 M_i \tag{23}$$

The response Eq. (22) can also be written as the sum of the individual modal contributions in the form

$$r = \sum^{p} c_i\, \psi_i \tag{24}$$

where the quantity c is given by

$$c_i = D_{ii}\, \psi_i^t\, R_o \tag{25}$$

The advantage in writing the response in the form of Eq. (24) will be apparent in Sec. V while discussing the procedure for selection of significant modes.

In the dynamic response [Eq. (22)], the time integration is involved only in the dynamic load factor matrix D. The elements D are essentially the convolution of the impulse response and the forcing function $F(t)$. Since the impulse response is easy to determine the complexity of evaluating the integrals is entirely dependent on the nature of the forcing function. However, if $F(t)$ can be represented numerically the load factor matrix can be determined by numerical integration.

III. Optimality Criteria for Dynamic Loads

A stiffness related optimality criteria is established in this section. The objective in this case is to minimize the weight of the structure while satisfying the dynamic stiffness requirements. In the derivation the configuration of the structure is assumed to be fixed and the sizes of the elements are treated as variables. The structure is discretized into n finite elements. The weight of the structure may be written as

$$W = \sum^{n} \rho_i\, A_i\, \ell_i \tag{26}$$

where $A_i\ell_i$ is the volume of each element and ρ_i is the mass density of the material. The dynamic stiffness of the structure in the j^{th} mode is defined by the Rayleigh quotient as

$$Z = \omega_j^2 = (\psi_j^t K \psi_j / \psi_j^t\, M\, \psi_j) \tag{27}$$

If Z is assumed to be the desired stiffness requirement then a function ϕ consisting of the weight and the stiffness requirement can be written as

$$\phi = \sum^{n} \rho_i A_i \ell_i + (1/\lambda) Z \tag{28}$$

where λ is the undetermined Lagrangian multiplier. Minimization of ϕ with respect to the design variables gives the condition for the stationary value of the weight with constraint condition Z as

$$(\partial\phi/\partial A_i) = \rho_i \ell_i + (1/\lambda)(\partial/\partial A_i) Z = 0 \tag{29}$$

Substitution of Eq. (27) into (29) gives the expression for the Lagrangian multiplier in the form

$$\lambda = (e_i/\psi_j^t M \psi_j) \tag{30}$$

where e_i is the ratio of the difference in strain and kinetic energy densities to the mass density of the i^{th} element in the given mode. It is given by

$$e_i = (\psi_j^t K_i \psi_j - \omega_j^2 \psi_j^t M_i \psi_j / \rho_i A_i \ell_i) \tag{31}$$

where K_i and M_i are the stiffness and mass matrices of the ith element in the structure coordinate system. Since the Lagrangian multiplier λ is constant Eq. (30) can be true only when all the elements of the structure have the same e in the given mode. This means that a structure vibrating in one of the natural modes has optimum distribution of material if the quantity e is same in all its elements.

Application of these optimality criteria for designing in the dynamic mode involves certain approximations. When the dynamic mode (response) primarily consists of a single natural mode, the optimality criteria is valid without approximation. However, if the dynamic mode representation requires more than one natural mode [Eq. (24)], then the optimality criteria is valid only approximately. The approximation results from the assumption that the quantity

$$(\partial/\partial A_i)[r^t] \ [K r - \omega^2 M r] \simeq 0 \tag{32}$$

where ω^2 is the Rayleigh quotient in the dynamic mode.

The optimality criteria derived so far is valid for single stiffness requirement. The extension of this to multiple stiffness requirements is

$$\sum_{j=1}^{p} (e_i^j/\lambda_j) = 1 \tag{33}$$

where p is the number of design conditions. The quantities e_i^1 $e_i^2 ... e_i^p$ are the ratios of the difference in strain and kinetic energy density to the mass density of the ith element in the respective design condition. The Lagrangian multipliers $\lambda_1 ... \lambda_p$ may be considered as weighting factors. These weighting factors are in general not a unique set and can be determined only approximately in an iterative procedure.

IV. Design in the Dynamic Mode

In general, the optimality criterion derived in Sec. III can be satisfied only by iteration. The necessary recursion relation for iteration is derived in Sec. IV. The optimality criterion when extended to the dynamic mode may be written as

$$\lambda = \frac{1}{r^t M r} \frac{[r^t K_i r - \omega^2 r^t M_i r]}{\rho_i A_i \ell_i} \tag{34}$$

where r is the dynamic mode [see Eq. (24)] and ω^2 is the Rayleigh Quotient in the dynamic mode. The design variable vector A is written as

$$A = \Lambda \alpha \tag{35}$$

where Λ is the normalizing or scaling parameter and α is the relative design variable vector. Multiplying the numerator and the denominator on the right-hand side of Eq. (34) by Λ^2, we can rewrite Eq. (34) as

$$\Lambda^2 = \frac{1}{\lambda r^t M r} \frac{[r^t K_i r - \omega^2 r^t M_i r]\Lambda^2}{\rho_i A_i \ell_i} \tag{36}$$

Multiplying both sides of Eq. (36) by α_i^2 and taking the square root the expression for the design variable may be written as

$$\alpha_i \Lambda = C \ \alpha_i [u_i'/\tau_i']^{1/2} \tag{37}$$

where C, u_i', and τ_i' are given by

$$C = [1/\lambda r^t M r]^{1/2} \tag{38}$$

$$u_i' = [r^t K_i r - \omega^2 r^t M_i r]\Lambda \tag{39}$$

$$\tau_i' = \rho_i \alpha_i \ell_i \tag{40}$$

Since α_i exists on both sides of Eq. (37), the recursion relation may be written as

$$(\alpha_i \Lambda)_{\nu+1} = C(\alpha_i)_\nu [u_i'/\tau_i']_\nu^{1/2} \tag{41}$$

where ν refers to the cycle of iteration. In Eq. (41), C is an arbitrary constant and need not be determined.[3] In case of multiple design conditions, the recursion relation may be written as

$$(\alpha_i \Lambda)_{\nu+1} = (\alpha_i)_\nu \left[\sum_{j=1}^{p} C_j (u_i^{(j)\prime}/\tau_i') \right]_\nu^{1/2} \tag{42}$$

Where $C_1 ... C_p$ are weighting constants in individual design conditions.

A step-by-step procedure for using Eq. (41) [or Eq. (42)] in an iterative alogrithm is as follows: 1) With assumed sizes for the elements the eigenvalues and eigenvectors of the structure are determined Eq. (16). An approximate procedure for determining the number of significant modes is given in Sec. III. 2) The dynamic mode (or response) is determined by Eqs. (22) or (24). 3) The dynamic stresses are determined from the dynamic mode. 4) The dynamic stress requirements are satisfied by scaling the sizes of the elements. The implications of scaling in various situations is discussed later in Sec. IV. 5) The structural elements are resized by using Eq. (41) [or Eq. (42)]. 6) Steps 1-5 are repeated so long as the design improves.

The implications and limitations of the scaling procedure for static cases is discussed in Refs. 2 and 3. In the design for dynamic loads the scaling procedure has additional limitations depending on the nature of the mass distribution on the structure. The implications of scaling in the two extreme cases of mass distribution are considered next.

Case 1: The structure has no attachments. the structural mass is the main contribution to the mass matrix. In addition, the change in the sizes of the elements produces a proportional change in the mass and stiffness matrices. In this case scaling of the structure does not affect the eigenvalues and modes. The shape of the dynamic mode is, therefore, not altered by scaling and the dynamic stress requirements can be satisfied by scaling just as in the static case.

Case 2: The contribution of the nonstructural mass is dominant and in comparison the structural mass can be ignored. In this case the eigenvalues of the structure increase in proportion to the scaling parameter and the dynamic mode needs updating with each change. However, there is no need for new eigenvalue analysis.

When the actual case does not belong to these extreme cases, additional eigenvalue analyses are necessary to update the dynamic mode, and the scaling has to proceed in small steps.

V. Significant Modes and Design Examples

Before presenting the design examples, an approximate procedure for determining the significant modes is presented here. The method is based on a study of the virtual work of the peak dynamic forces and the dynamic mode as given by Eq. (24). The virtual work of the peak forces is defined as

$$V_w = R_o' \, r \tag{43}$$

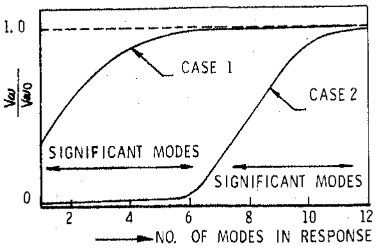

Fig. 1 Significant modes.

Figure 1 illustrates two possible cases of the variation of the virtual work V_w with the number of modes included in the dynamic response. In the first case, the 5 modes at the lower end of the spectrum are significant. In the second case, modes 7-11 are significant, and the remaining modes can be left out. For a given design, the dynamic response is determined by adding the effect of one mode at a time in Eq. (24). After each addition, the virtual work V_w is determined from Eq. (43). A plot of the virtual work against the number of modes would reach a plateau when an adequate number of modes are included in the response. This procedure for determining the significant modes is recommended before starting the iteration procedure discussed in Sec. IV.

A wing and an arch are the two examples discussed in Sec. V. The arch example has 2 cases with different load distribution. In all cases, the initial conditions are assumed to be zero. The dynamic mode [Eqs. (22) or 24)] is determined by considering first 15 modes of the structure in both examples.

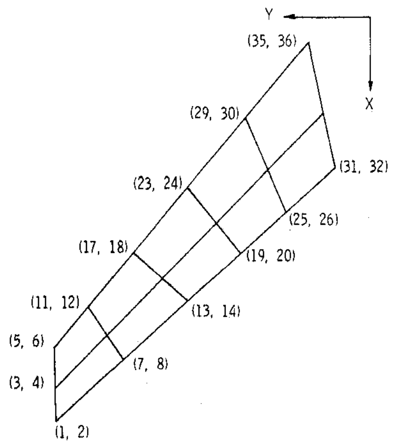

Fig. 2 Wing.

Wing Structure

A three-spar wing structure is idealized by 155 bar elements (each rectangular panel is made up of two crossbars in addition to the bars on the edges). The plan form of the wing is shown in Fig. 2 and the node coordinates and the forces are given in Table 1. The leading and trailing edges are subjected to impulse in the z-direction, represented by $R_o\delta(t)$. The elements of R_o are given in Table 1. The magnitudes of the forces are different at different nodes but they are applied at the same instant. The material is assumed to be aluminum with the maximum dynamic stress limited to 25000 psi. The

Table 1 Coordinates and loads on the wing structure

			Z in.		Loading (kips) *Z*-component	
Node nos.	*X* in.	*Y* in.	Top node	Bottom node	Top node	Bottom node
1, 2	249.06	160.00	2.279	−2.265	+42.0	+27.0
3, 4	236.14	160.00	2.218	−2.223		
5, 6	217.91	160.00	1.816	−1.838	−13.0	−8.0
7, 8	224.89	131.33	3.460	−3.417	+8.0	+5.0
9, 10	215.05	137.86	3.098	−3.081		
11, 12	202.23	145.83	2.286	−2.214	−13.0	−9.0
13, 14	203.08	105.47	4.494	−4.383	+10.0	+6.0
15, 16	193.59	115.33	3.985	−3.975		
17, 18	181.91	127.46	2.880	−2.924	−4.0	−3.0
19, 20	183.63	82.40	5.420	−5.297	+4.0	+2.0
21, 22	171.76	92.41	4.892	−4.859		
23, 24	156.95	104.89	3.613	−3.662	+1.0	+1.0
25, 26	167.65	63.45	6.147	−6.031	+2.0	+1.0
27, 28	151.06	70.67	5.744	−5.691		
29, 30	129.31	79.92	4.433	−4.481	+2.0	1.0
31, 32	151.67	44.50	7.059	−7.047		
33, 34	130.37	48.94	6.619	−6.549		
35, 36	101.66	54.95	5.260	−5.307		

Table 2 Wing weight and frequencies

Cycle	Wt	Frequencies (cps)									
	lb	1	2	3	4	5	6	7	8	9	10
1	3258.3	19.7	28.1	66.9	81.3	81.8	96.9	130.6	161.8	183.9	196.8
2	1351.7	34.4	36.1	86.8	88.7	105.2	109.9	178.6	202.1	211.0	248.9
3	1333.6	34.5	46.4	87.4	89.2	95.3	138.2	203.7	209.3	231.6	235.4

Table 3 Values of V_w / V_{wo} with number of modes in response wing

Modes cycle	1	2	3	4	5	6	7	8	9	10
1	0.6553	0.6553	0.9151	0.9347	0.9349	0.9349	0.9885	0.9896	0.9896	0.9982
2	0.4696	0.4708	0.8478	0.8484	0.8484	0.8497	0.9503	0.9571	0.9571	0.9571
3	0.0000	0.3644	0.8680	0.8685	0.8685	0.8686	0.8686	0.9621	0.9634	0.9634

Table 4 Arch weight and frequencies: case 1

Cycle	Wt	Frequencies (cps)									
	lb	1	2	3	4	5	6	7	8	9	10
1	1231.2	155.8	199.1	318.2	462.8	569.2	737.2	897.6	1012.0	1305.0	1315.0
2	996.5	161.8	209.2	341.6	500.4	604.4	751.1	918.9	1025.0	1324.0	1324.0
3	972.2	165.0	212.8	349.1	505.5	608.6	751.1	925.9	1018.0	1324.0	1324.0

Table 5 Values of V_w / V_{wo} with number of modes in response (arch, case 1)

Modes Cycle	1	2	3	4	5	6	7	8	9	10
1	0.9791	0.9791	0.9791	0.9972	0.9993	0.9993	0.9993	0.9996	0.9998	0.9998
2	0.9766	0.9766	0.9766	0.9964	0.9995	0.9995	0.9995	0.9995	0.9999	0.9999
3	0.9763	0.9763	0.9763	0.9964	0.9995	0.9995	0.9995	0.9998	0.9998	0.9999

weight of the structure satisfying the dynamic stress requirement is 3258 lb in the first cycle. After two iterations [using Eq. (41)] the weight is reduced to 1334 lb. Table 2 gives the values of the first 10 frequencies in 3 iterations.

The data given in Table 3 reveals important information about the significance of various modes of the structure. It is interesting to note that the first mode has contribution of about 66% to the dynamic response in the first cycle while reducing to zero in the third cycle. After resizing twice the first two modes switched their roles from bending to inplane (x-y plane) modes. The external loading produces primarily bending and some twist and as a result inplane modes do not contribute to the dynamic response. In the first cycle primary contribution is from the first mode followed by about 27% from the third mode. The remaining modes have no significant contribution. After the first resizing contribution from modes 1 and 3 became equal and the 7th contributed about 10%. In the third cycle contribution from the first mode disappeared completely while participation of the second mode increased from 0-36%. This mode switching can cause interesting problems to automated design schemes. On practical structures when the nonstructural mass attachments are added to the wing, the switching of modes may not be a serious problem.

Circular Arch

The circular arch shown in Fig. 3 is represented by segmented beam elements. The principal moment of inertia is considered as the design variable for the beams. It is assumed that the radius of gyration of the beam section is the same for all the elements and is not allowed to change during resizing. A sandwich beam with constant depth and variable face sheet thicknesses would conform to this assumption. The material of the arch is assumed to be steel with maximum dynamic stress limited to 29000 psi.

In the first case the structure is subjected to impulse force extending over the entire structure with the distribution shown in Fig. 3 and $P_o = 100K$. The design started with uniform sizes for all elements and the weight of the structure satisfying the dynamic stress requirements is 1231.2 lb. After resizing two times, the weight reduced to 972.2 lb. For this loading, the first mode is the most significant mode with 97.63% contribution followed by the fourth mode with less than 2% contribution. (See Tables 4 and 5 for details.)

In the second case, the arch is subjected to a radial force of 200K at the quarter point. The remaining design conditions are same as in the first case. The initial weight of the structure (uniform sizes) is 572.00 lb. For this loading also the major contribution to modal response is from the first mode but not as large as in the first case. At least four more modes have some contribution. In both arch problems, there is practically no mode switching due to resizing. The reason for this behavior is that the natural frequencies of the arch are better separated than the wing frequencies. (For details see Tables 6-9.) These 2 examples show that the spatial distribution of the external force plays an extremely important role in determining the response of the structure and an accurate description of it is vital for the validity of the final design even in the dynamic case.

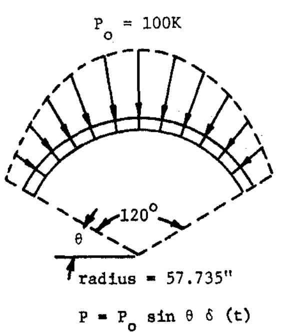

Fig. 3 Circular arch.

Table 6 Case 1 moment of inertia of arch members (in.^{4}) (radius of gyration = 7.4575)

Cycle Memb.	1	2	3
1, 12	2004	2166	2253
2, 11	2004	1669	1700
3, 10	2004	1375	1312
4, 9	2004	1374	1262
5, 8	2004	1508	1414
6, 7	2004	1638	1551

Table 7 Arch weight and frequencies: case 2

Cycle	Wt lb	Frequencies (cps) 1	2	3	4	5	6	7	8	9	10
1	572.0	155.8	199.1	318.2	462.8	569.2	737.2	897.6	1012.0	1305.0	1315.0
2	396.0	158.2	215.3	320.2	428.0	587.0	797.8	925.9	1056.0	1334.0	1344.0
3	350.0	161.8	227.0	333.9	439.9	600.1	804.2	932.8	1044.0	1305.0	1344.0
4	310.5	165.0	235.3	341.6	448.6	608.6	810.6	932.8	1025.0	1285.0	1343.0
5	291.9	168.1	239.7	347.2	451.5	612.8	816.9	925.8	1006.0	1275.0	1334.0

Table 8 Values of V_w / V_{wo} with number of modes in response (arch, case 2)

Modes / Cycle	1	2	3	4	5	6	7	8	9	10
1	0.4127	0.6434	0.8632	0.9207	0.9764	0.9832	0.9873	0.9886	0.9907	0.9907
2	0.5496	0.6800	0.8576	0.8997	0.9745	0.9806	0.9915	0.9929	0.9935	0.9936
3	0.5489	0.6693	0.8344	0.8767	0.9682	0.9747	0.9889	0.9910	0.9916	0.9916
4	0.5154	0.6402	0.8090	0.8559	0.9631	0.9697	0.9859	0.9889	0.9894	0.9896
5	0.4825	0.6114	0.7840	0.8355	0.9582	0.9654	0.9823	0.9861	0.9866	0.9868

Table 9 Case 2 moment of inertia of arch members (in.^{4}) (radius of gyration = 7.4575)

Cycle / Memb	1	2	3	4	5
1	931	1437	1701	1794	1794
2	931	645	748	828	904
3	931	744	544	393	314
4	931	1027	893	7 4	632
5	931	737	659	548	481
6	931	406	353	284	244
7	931	398	284	223	187
8	931	486	327	252	209
9	931	522	355	265	219
10	931	508	365	272	224
11	931	444	334	259	215
12	931	387	282	218	181

VI. Summary and Conclusions

The optimality criteria approach is extended to automated design of minimum weight structures subjected to impulse type loading. The optimality criterion derived for natural modes was used as an approximation to design in the dynamic mode. The dynamic mode is defined as being a combination of a set of natural modes. A simple recursion relation is presented for resizing of the structure. A step-by-step procedure for the design is outlined. The recursion relation in conjunction with a scaling procedure was effectively used in designing a wing idealized with bar elements and a circular arch with beam elements.

The attractive features of the method are its simplicity and computational efficiency. The resizing algorithm requires little more than the information generated in an eigenvalue analysis. The number of cycles of iteration is independent of the number of variables in the structure and they seldom exceed five or six. The applications in this paper should be considered as being preliminary. Extensive numerical experimentation is necessary to study the convergence characteristics, effect of mode switching, the role of nonstructural mass attachments and applications to high-frequency vibration problems.

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